A SynopsisOn

"Analytical Solution of Fractional Differential Equations by Some Improved Methods"

Submitted to

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For the Degree of

Doctor of Philosophy (Ph.D.)

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1. Introduction:

Fractional calculus and fractional differential equations have emerged as the most important and prominent areas of interdisciplinary interest. Fractional calculus has a history of more than 300 years, yet its applicability in different domains has been realized only from the past three decades. [1-3] This is due to the intensive development of the theory of fractional calculus itself as well as its applications. Apart from diverse areas of pure mathematics, fractional differential equations can be used in modeling of various fields of science and engineering. In the recent years, fractional calculus has played a very important role in various fields such as mechanics, electricity, chemistry, biology, economics, notably control theory, and signal and image processing. Major topics include anomalous diffusion, vibration and control, continuous time random walk, Levy statistics, fractional Brownian motion, fractional neutron point kinetic model, power law, Riesz potential, fractional derivative and fractals, computational fractional derivative equations, nonlocal phenomena, history-dependent process, porous media, fractional filters, etc. Due to non-local property of the fractional differential equations, the non-linear systems like the non-linear oscillations of earthquake [5], seepage flow in porous media [4] and traffic flow in fluid dynamics and viscoelastic problems are modeled in to fractional differential equations [1, 6] studied.

Among the investigations for fractional differential equations, the research for seeking analytical solutions and numerical solutions of fractional differential equations is an important topic.

2. Review of Literature and Development in the Subject:

Today there exist many different forms of fractional operators. However, there are many definitions in the literature nowadays, but few of them are commonly used, including Riemann-Liouville [1], Caputo [1, 9], Weyl [1], Jumarie [17, 18], Hadamard

[10], Davison and Essex [10], Riesz [1], Erdelyi-Kober [1], and Coimbra [11], etc. but the Riemann- Liouville Operator is still the most frequently used when fractional integration is performed. Most of nonlinear fractional differential and integral equations do not have analytical solution, so approximations and numerical techniques have to be used to solve fractional differential and integral equations. The decomposition method and variational iteration method [7,8] have been extensively studied. Apart from these semi-analytic iterative methods, few numerical methods have been developed till now.

Recently, new methods for solving Fractional Differential Equations have been developed in the literature. Many powerful and efficient methods have been proposed to obtain numerical solutions and exact solutions of fractional differential equations so far. For example, these methods include the Adomian decomposition method [12, 13], the variational iterative method [14], the finite difference method [15], etc. Based on these methods, a variety of fractional differential equations have been investigated. S. Zhang and H.-Q. Zhang [16] proposed a new direct algebraic method named fractional subequation method for solving fractional differential equations based on the homogeneous balance principle, the modified Riemann-Liouville derivative by Jumarie [17, 18], and the fractional Riccati equation.

Latterly, a new iterative method was presented by Daftardar – Gejji and Jafari. This technique solves many types of nonlinear equations such as ordinary and partial differential equations of integer and fractional order. This method to obtain the solution of linear /non-linear diffusion and wave fractional equations. Daftardar – Gejji and Bhalekar used it for solving fractional boundary value problems and evolution equations.

3. Aim and Objectives of Research:

1) To study Some Improved Methods to Solve Fractional Differential Equations.
2) To obtain analytical solutions of fractional differential equations using different improved methods and will compare their solutions, with solutions obtained by other methods, if it is possible.

3) To stimulate the solutions of fractional partial differential equations using Mathematical software Mathematica, MATLAB to discuss the nature of the solutions.
4) To prove some basic results of fractional derivatives and differential equations by our own way.
5) To improve few methods or innovate new methods to solve fractional differential equations and we will solve some counter examples.
6) To publish research papers in reputed journals.
7) We will try to solve Fractional differential equations with mixed partial derivatives and System of Fractional differential equations with mixed partial derivatives.

4. Methodology:

1) Relevant knowledge of the Fractional derivatives and Fractional differential equations will be obtained by the survey of research papers, review articles and journals. Also, by referring the relevant reference books on it.
2) We will study Improved Analytical methods for obtaining solutions of fractional differential equations.
3) We will try to represent solutions of fractional differential equations graphically by Mathematica and discuss their nature.
4) We will visit different research institutes like Savitribai Phule Pune University, Pune, Shivaji University, Kolhapur, Dr Babasaheb Ambedkar Marathwada University, Aurangabad, University of Hydrabad, etc to interact with eminent Mathematicians doing work in Fractional Differential equations. Also, would like to attend national, international conference related to my research work.

5. Importance of study:

We hope that the present work will be useful for scientists, engineers, researchers, mathematicians in their endeavors to analyze the non-linear problems of universe in a better way. It has better scope for further development in future. The fractional

differential equation is useful in many of the fields like in the viscoelastic materials, polymer materials, acoustic wave propagation in inhomogeneous porous material, fluid flow, geology, optics etc... Also useful in engineering field like chemical engineering, quantum mechanics etc. The theory of fractional calculus (FC), with chaos and dissipation construction has been the primary sign of current nonlinear science.

6. Plan of Research Work:

Sr.No	Period	Plan of the work	Targets to be achieved
1	First year	Study of the background required for the research topic. Collecting the papers, analyze them.	Good command on the subject and clear the course work. Try to get some Construction techniques
2	Secon dyear	Work on some problems to be included in the thesis.	Submit at least two papers.
3	Third Year	Work on some problems to be included in the thesis.	Submit at least two papers.
4	Fourth Year	Publishing some papers and submitting the Thesis.	Some more papers and Thesis submission.

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